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Tachyon condensation in open-closed p-adic string theory

Nicolas Moeller and Martin Schnabl

Center for Theoretical Physics, Massachusetts Institute of Technology
Cambridge, MA 02139, U.S.A.
E-mail: moeller@lns.mit.edu, schnabl@lns.mit.edu

Abstract: We study a simple model of p-adic closed and open strings. It sheds some light on the dynamics of tachyon condensation for both types of strings. We calculate the effect of static and decaying D-brane configurations on the closed string background. For closed string tachyons we find lumps analogous to D-branes. By studying their fluctuation spectrum and the D-branes they admit, we argue that closed string lumps should be interpreted as spacetimes of lower dimensionality described by some noncritical p-adic string theory.

Keywords: Tachyon Condensation, String Field Theory.

Contents

1. Introduction

The study of open p-adic string theory [1, 2] has revealed that this theory possesses many nonperturbative features that are common to open string field theory; yet p-adic string theory is a very simple model whose solutions, in some cases, can be found analytically. Besides its perturbative vacuum, it has a true vacuum in which there is no solution to the linearized equations of motion,[1] a situation that is believed to happen in open string field theory, where the open string excitations should disappear once the D-brane condenses [3]. P-adic string theory also possesses solitonic lump solutions [1, 4, 5] that can be interpreted as lower dimensional D-branes. This theory is thus a nice model for open string tachyon condensation.

An interesting question is whether one could extend these p-adic models to study the condensation of closed string tachyons as well. Closed p-adic string amplitudes have been studied in [6], and a closed p-adic lagrangian was derived in [7]. This lagrangian, however, looks exactly like an open p-adic lagrangian, only with a different tachyon mass. And if one takes this model seriously, one would have to give an interpretation for the closed string "D-branes" of any dimension, that exist in this theory. Other difficulties related to closed strings were pointed out in [8]. In particular it was shown that one-loop diagrams of open p-adic string theory do not possess any pole that could correspond to massless closed strings.

[1] Anharmonic oscillations in the true vacuum were found in [13].

However, in [2] Brekke and Freund presented a very interesting mixed lagrangian of both open and closed p-adic tachyons, that was derived from the p-adic n-point amplitudes for open and closed tachyons. Surprisingly, we haven't found any study of this lagrangian in subsequent literature. It is the object of this paper to study this open and closed p-adic theory.

Although the pure open and pure closed p-adic string theories behave similarly, we will see that the two fields have in fact very distinct dynamics in the mixed theory. We will exhibit the fundamental differences between both fields and show how this leads us to interpret the lumps of the closed tachyon as lower dimensional spacetimes. In particular, we will see that this picture is coherent since we cannot have D-branes with more worldvolume dimensions than the dimension of spacetime. We further support our claims by studying the fluctuations of closed string lumps. All fluctuations are strictly normalizable, which means that all excitations are confined to the worldvolume of the lump. Far away from the core of the lump, the closed string tachyon is in its true vacuum, and one can check that the linearized equations of motion do not admit any solutions. The fluctuation spectrum is equidistant, strongly suggesting that the full theory around these lumps should be described by some sort of lower-dimensional noncritical string theory. We are tempted to speculate that similar lumps should exist in ordinary string theories, in which case one would get localization to lower dimensions of all physics including the gravity.

Another domain of application of open-closed p-adic string theory is to study the effect of static or decaying D-branes on the ambient spacetime. Again, the ambient spacetime in this theory is fully specified by the closed string tachyon background. Of particular interest are the time dependent solutions [9]–[12], which have recently attracted a lot of attention and whose physics has not yet been fully understood. One of the most pressing problems is that the approximations of the open cubic string field theory considered so far [13]–[16] failed to reproduce the results obtained by conformal field theory arguments. Instead of the open string tachyon relaxing in the true vacuum, the authors found oscillations with ever growing amplitude. It is not clear whether this is caused by the omission of closed strings, by the crude approximation taking into account the tachyon field only, or whether it reflects just a wrong choice of variables.

One possibility is that the string field theory calculations can be reconciled with CFT by taking into account that at finite string coupling the rolling open string tachyon dissipates its energy into closed strings [17, 18, 19, 20, 21, 24]. We study this process in our theory, but are unable to reach definite conclusions. One of the reasons is that in the interacting theory of closed and open strings it is impossible to separate the total conserved energy into open and closed string parts.

This paper is structured as follows: First, in section 2, we study the vacuum structure of the open-closed p-adic theory. In section 3, we study lumps of both the open and closed fields, and we discuss how this leads us into our interpretation of closed lumps as lower dimensional spacetimes. Section 4 is devoted to a study of the back-reaction of open lumps on the flat space background. In section 5 we extend those results to rolling solutions. Finally, section 6 is devoted to discussions and conclusions.

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2. Open-closed p-adic string theory and its vacuum structure

The full action for the coupled p-adic open and closed strings was calculated in [2] by requiring that it reproduces correctly all the n-point p-adic tachyon amplitudes involving both open and closed tachyons. It is given by $S = \int d^D x \mathcal{L}$, where the lagrangian is

$$\begin{aligned} \mathcal{L} = & -\frac{1}{2g^2}\frac{p^2}{p-1}\phi p^{-\Box/2}\phi - \frac{1}{2h^2}\frac{p^4}{p^2-1}\psi p^{-\Box/4}\psi + \\ & +\frac{1}{h^2}\frac{p^4}{p^4-1}\psi^{p^2+1} + \frac{1}{g^2}\frac{p^2}{p^2-1}\psi^{p(p-1)/2}\left(\phi^{p+1}-1\right), \end{aligned} \tag{2.1}$$

where ϕ and ψ are respectively the open and closed tachyons, $\Box = -\partial_t^2 + \nabla^2$ is the D-dimensional d'Alembertian, and g and h are respectively the open and closed string coupling constants.[2] The dimension D of spacetime can be chosen arbitrarily in this model. Note that in order to derive this lagrangian, p was assumed in [2] to be a prime integer greater than two. But the lagrangian itself makes sense for any integer p greater than one. In fact, it even makes sense in the limit $p \to 1$.

Let us look first at the vacuum structure of the theory. The potential is

$$V = \frac{1}{h^2}\left[\frac{1}{2}\frac{p^4}{p^2-1}\psi^2 - \frac{p^4}{p^4-1}\psi^{p^2+1} + \lambda^2\left(\frac{1}{2}\frac{p^2}{p-1}\phi^2 - \frac{p^2}{p^2-1}\psi^{p(p-1)/2}\left(\phi^{p+1}-1\right)\right)\right]. \tag{2.2}$$

We have introduced here a parameter $\lambda = h/g$, which governs the interaction between the open and closed string sectors.[3] In real string theory $h \sim g^2$ up to some numerical proportionality factor. We assume this to be the case also in p-adic string theory, though we shall leave the proportionality factor undetermined. We thus find that $\lambda \sim g$ has the interpretation of the open string coupling constant.

The stationary points of the potential are given by the equations

$$\psi\left(1 - \psi^{p^2-1} - \lambda^2\frac{p-1}{2p}\psi^{\frac{p(p-1)}{2}-2}\left(\phi^{p+1}-1\right)\right) = 0 \tag{2.3}$$

$$\phi\left(1 - \phi^{p-1}\psi^{\frac{p(p-1)}{2}}\right) = 0 \tag{2.4}$$

which have the following interesting solutions:

- For all p there is a local maximum solution $\phi = \psi = 1$, this is the perturbative vacuum for both sectors. We may interpret it as a D-brane filling all of our flat spacetime.

- For $p > 2$ there is always a local minimum solution with $\phi = \psi = 0$, which is supposed to be the true open and closed string vacuum. For $p = 2$ this vacuum gets shifted by the interaction to $\phi = 0$, $\psi = -\frac{\lambda^2}{4} + O(\lambda^8)$.

[2] We are using units in which $\alpha' = \frac{1}{2}$.

[3] One may wish to rescale the fields as $\hat{\phi} = \phi/g$ and $\hat{\psi} = \psi/h$ so that the kinetic terms are independent of the coupling constants. We find this quite inconvenient since this would make the position of the perturbative vacuum $(\phi, \psi) = (1, 1)$ coupling constant dependent as well as the profile of some other classical solutions. Of course all physical results are independent of the chosen normalization.

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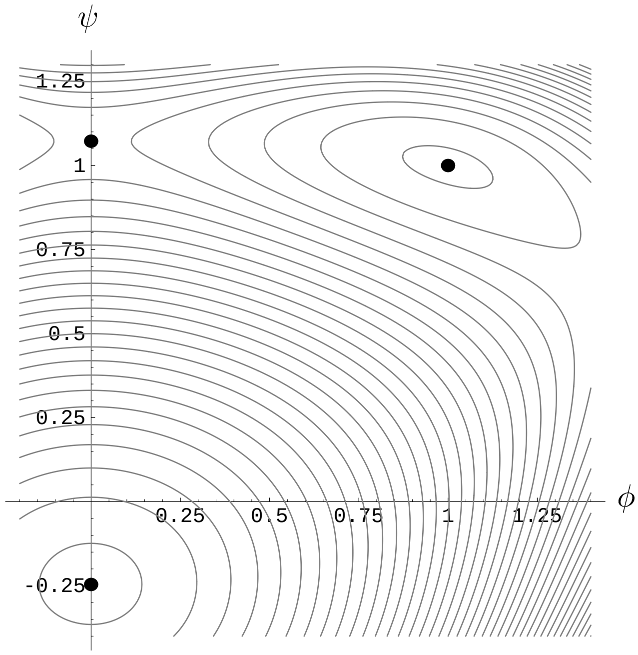

Figure 1: Equipotential curves for $p = 2$ and $\lambda = 1$. The dots mark the positions of the local minimum $(0, -0.246)$ which is the true vacuum of the theory, the saddle point $(0, 1.07)$ which is interpreted as a flat spacetime with no D-brane, and the local maximum $(1, 1)$ corresponding to a D-brane filling all of spacetime.

- A third interesting solution is the saddle point $\phi = 0$, $\psi = 1 + \frac{\lambda^2}{2p(p+1)} + O(\lambda^4)$. It is in fact a local minimum in the ϕ direction and a local maximum in the ψ direction. And it is thus interpreted as being a perturbative flat closed string background with no D-brane.

The positions of these three extrema are shown in figure 1 for the case $p = 2$ and $\lambda = 1$. Finally let us note that there is a couple of saddle-point solutions, which come from infinity as the coupling λ is turned on. We won't be interested in these solutions here.

From this extrema analysis, we can already make an attempt to interpret the true vacuum of ψ as being a configuration in which there is no spacetime. Indeed, we note that there is no solution in the vicinity of $\psi = 0$ and $\phi = 1$, which could be a true vacuum for the closed string and a perturbative vacuum for the open string. This is very natural from the physical point of view, since such a solution would represent a D-brane living outside spacetime. We will thus adopt this interpretation, and address it further in section 3.

Finally, note that the lagrangian (2.1) differs from the one in [2] by a constant term. Indeed, the authors of [2] chose to set the value of the potential to zero at the local maximum (perturbative vacuum) $\phi = \psi = 1$. We don't make this choice here; in fact, for all $p \neq 2$, the potential (2.2) is zero at the local minimum $\phi = \psi = 0$.

3. Lower-dimensional spacetimes and double lumps

In this section, we will argue that lumps of the closed tachyon ψ represent flat spacetime extended along the lump's worldvolume only. Those are therefore spacetimes of lower dimensions.

Let us first write the equations of motion derived from (2.1)

$$p^{-\frac{1}{4}\Box}\psi = \psi^{p^2} + \lambda^2\,\frac{p-1}{2p}\,\psi^{\frac{p(p-1)}{2}-1}\left(\phi^{p+1}-1\right)$$
$$p^{-\frac{1}{2}\Box}\phi = \phi^p\,\psi^{\frac{p(p-1)}{2}}\,. \tag{3.1}$$

For finite λ, these equations are very hard to solve analytically. We will therefore start our discussion by setting $\lambda = 0$. Recall that λ is proportional to the string coupling. In the limit $\lambda = 0$ the first equation in (3.1) determines a classical closed string background, the second equation then tells the field ϕ how to propagate in that background. The back-reaction is neglected in this limit. For fields depending only on the d spatial coordinates x the equations then take the form

$$p^{-\frac{1}{4}\partial_x^2}\psi = \psi^{p^2} \tag{3.2}$$
$$p^{-\frac{1}{2}\partial_x^2}\phi = \phi^p\,\psi^{\frac{p(p-1)}{2}}\,. \tag{3.3}$$

Let us shortly describe the case when ψ sits on its unstable vacuum. If we set $\psi = 1$, the equation (3.2) is trivially satisfied, and equation (3.3) becomes exactly the static equation of motion of open p-adic string theory, which we know admits gaussian lump solutions [1, 4, 5]. We thus interpret these solutions as lower dimensional D-branes living in flat spacetime. By flat spacetime we mean more precisely that the closed string tachyon is in its perturbative vacuum.

We now want to consider nontrivial static configurations of ψ. We readily see that (3.2) looks very much like the static equation of motion of purely open p-adic string theory. And therefore we know that it must have lump solutions. Indeed, using the identity

$$e^{-a\partial^2}e^{-bx^2} = (1-4ab)^{-\frac{d}{2}}e^{-\frac{b}{1-4ab}x^2} \tag{3.4}$$

we find the codimension-d lump solution

$$\psi_0(x) = p^{\frac{d}{p^2-1}}\,e^{-\frac{p^2-1}{p^2\log p}x^2}\,. \tag{3.5}$$

Far from the center of the lump $x = 0$ the closed tachyon $\psi_0(x)$ rapidly approaches its true vacuum $\psi = 0$. Assuming that $\phi_0(x)$ has finite asymptotic value and its derivatives vanish asymptotically, it is obvious from equation (3.3) that the open string tachyon itself must approach its true vacuum. It follows then, that if we are going to find any D-brane in this background, its dimensionality has to be smaller or equal to that of the closed tachyon lump.

In fact, such solutions can be easily found. Plugging (3.5) in (3.3) and using a gaussian ansatz we find for example

$$\phi_0(x) = p^{\frac{d(p+2)}{2(p^2-1)}}\,e^{-\frac{p^2-1}{2p^2\log p}x^2}\,. \tag{3.6}$$

This corresponds to a D-brane filling out our $D-d$ dimensional spacetime. Lower dimensional D-branes can be also easily found, but higher dimensional ones do not exist for the reasons explained above. Since both lumps (3.5) and (3.6) are sitting on top of each other we call this solution a *double lump*.

To test our interpretation of the double lump further, let us study the spectrum of fluctuations around it. For fluctuating closed and open string tachyon field we write

$$\begin{aligned}\psi(\vec{y},\vec{x}) &= \psi_0(\vec{x}) + \delta\psi(\vec{y},\vec{x})\,,\\ \phi(\vec{y},\vec{x}) &= \phi_0(\vec{x}) + \delta\phi(\vec{y},\vec{x})\,,\end{aligned}\tag{3.7}$$

where $\vec{x}$ and $\vec{y}$ are the transverse and parallel coordinates to the lump respectively. The part of the action quadratic in the fluctuations is then

$$S_f = -\frac{1}{2h^2}\frac{p^2}{p-1}\int d^{D-d}y\,dx^d\,(\delta\psi,\delta\phi)\begin{pmatrix}K_{\psi\psi} & K_{\psi\phi}\\ K_{\phi\psi} & K_{\phi\phi}\end{pmatrix}\begin{pmatrix}\delta\psi\\ \delta\phi\end{pmatrix}.\tag{3.8}$$

Here

$$\begin{aligned}K_{\psi\psi} &= \frac{p^2}{p+1}p^{-\frac14(\Box_y+\Box_x)} - \frac{p^4}{p+1}\psi_0^{p^2-1} + O(\lambda^2)\,,\\ K_{\psi\phi} &= K_{\phi\psi} = -\lambda^2\,\frac{p(p-1)}{2}\psi_0^{\frac{p(p-1)}{2}-1}\phi_0^p\,,\\ K_{\phi\phi} &= \lambda^2 p^{-\frac12(\Box_y+\Box_x)} - \lambda^2 p\psi_0^{\frac{p(p-1)}{2}}\phi_0^{p-1}\,.\end{aligned}\tag{3.9}$$

Following [5] we shall decompose the fluctuations as

$$\begin{aligned}\delta\psi(\vec{y},\vec{x}) &= \sum_{n_1,\ldots,n_d=0}^{\infty} u_{\vec{n}}(\vec{y})\,H_{n_1}(\alpha x_1)\ldots H_{n_d}(\alpha x_d)\,\psi_0(\vec{x})\,,\\ \delta\phi(\vec{y},\vec{x}) &= \sum_{n_1,\ldots,n_d=0}^{\infty} v_{\vec{n}}(\vec{y})\,H_{n_1}(\beta x_1)\ldots H_{n_d}(\beta x_d)\,\phi_0(\vec{x})\,,\end{aligned}\tag{3.10}$$

where $H_n(\xi)$ are the Hermite polynomials, normalized in such a way as to obey the orthogonality condition

$$\int_{-\infty}^{\infty} d\xi\,\exp(-\xi^2)H_m(\xi)H_n(\xi) = \pi^{\frac12}2^n n!\delta_{mn}\,.\tag{3.11}$$

Using the identity

$$H_n(\xi) = n!\oint\frac{dz}{2\pi i}z^{-n-1}e^{-z^2+2z\xi}\,,\tag{3.12}$$

it is quite easy to derive a generalization of the formula (3.4), which for $d=1$ reads

$$\begin{aligned}e^{-a\partial^2}H_n(\alpha x)e^{-bx^2} &= (1-4ab)^{-\frac12}\left(1+\frac{4a\alpha^2}{1-4ab}\right)^{\frac{n}{2}}\times\\ &\quad\times H_n\left(\frac{\alpha x}{\sqrt{(1-4ab)(1-4ab+4a\alpha^2)}}\right)e^{-\frac{b}{1-4ab}x^2}\,.\end{aligned}\tag{3.13}$$

Plugging the decomposition (3.10) into the action (3.8) and integrating over x, we end up with an action for the modes $u_{\vec{n}}(y)$ and $v_{\vec{n}}(y)$

$$S_f = -\frac{1}{2h^2}\frac{p^2}{p-1}\int d^{D-d}y \sum_{\vec{n},\vec{m}=0}^{\infty} (u_{\vec{n}}, v_{\vec{n}}) \begin{pmatrix} K^{uu}_{\vec{n}\vec{m}} & K^{uv}_{\vec{n}\vec{m}} \\ K^{vu}_{\vec{n}\vec{m}} & K^{vv}_{\vec{n}\vec{m}} \end{pmatrix} \begin{pmatrix} u_{\vec{m}} \\ v_{\vec{m}} \end{pmatrix}. \tag{3.14}$$

To find the spectrum of fluctuations we have to look at the zero modes of the kinetic term. Note that from (3.9) we have

$$\det \begin{pmatrix} K^{uu}_{\vec{n}\vec{m}} & K^{uv}_{\vec{n}\vec{m}} \\ K^{vu}_{\vec{n}\vec{m}} & K^{vv}_{\vec{n}\vec{m}} \end{pmatrix} = \det\left(K^{uu}_{\vec{n}\vec{m}}\right)\det\left(K^{vv}_{\vec{n}\vec{m}}\right) + O(\lambda^2)\,, \tag{3.15}$$

since to lowest order in λ^2 the determinant receives contributions only from the two blocks on the diagonal. Therefore to lowest order the mass spectrum is given by the zeros of the two determinants of $K^{uu}_{\vec{n}\vec{m}}$ and $K^{vv}_{\vec{n}\vec{m}}$, and is not affected by the presence of the terms which mix the two kinds of excitations.

The two constants α and β, which entered our fluctuation ansatz were arbitrary. For a convenient choice

$$\begin{aligned} \alpha &= \sqrt{\frac{p^4-1}{p^2\log p}}\,, \\ \beta &= \sqrt{\frac{p^4-1}{2p^2\log p}}\,, \end{aligned} \tag{3.16}$$

we find that the matrices $K^{uu}_{\vec{n}\vec{m}}$ and $K^{vv}_{\vec{n}\vec{m}}$ become diagonal in the mode space

$$\begin{aligned} K^{uu}_{\vec{n}\vec{m}} &= \frac{p^2}{p+1}\left(\prod_{i=1}^{d} p^{\frac{p^2+1}{p^2-1}}\sqrt{\pi\frac{p^2\log p}{p^4-1}}\,2^{n_i}n_i!\right)\left(p^{2\sum n_i-\frac{1}{4}\Box_y}-p^2\right)\delta_{\vec{n}\vec{m}}\,, \\ K^{vv}_{\vec{n}\vec{m}} &= \lambda^2\left(\prod_{i=1}^{d} p^{\frac{p^2+p+1}{p^2-1}}\sqrt{\pi\frac{2p^2\log p}{p^4-1}}\,2^{n_i}n_i!\right)\left(p^{2\sum n_i-\frac{1}{2}\Box_y}-p\right)\delta_{\vec{n}\vec{m}}\,. \end{aligned} \tag{3.17}$$

With the help of the approximate factorization (3.15) we find that the spectrum of fluctuations naturally splits into two series

$$\begin{aligned} m^2_{\delta\psi} &= 8(N-1) + O(\lambda^2)\,, \\ m^2_{\delta\phi} &= 4N-2 + O(\lambda^2)\,, \end{aligned} \tag{3.18}$$

where $N = n_1 + \cdots + n_d$ is the total oscillation number. It strongly resembles the spectrum of string theory. At zero level ($N = 0$) we have two tachyons with m^2 equal to -2 or -8. These are the open and closed string tachyon respectively. Then there are d massless modes, which for small but finite λ are just the Goldstone modes translating both lumps simultaneously in the transverse directions. There is obviously no Goldstone mode which could be associated with the open string spectrum and which would make the D-brane leave the spacetime it sits in. The higher level states in the spectrum have the same spacing as for the open or closed string.

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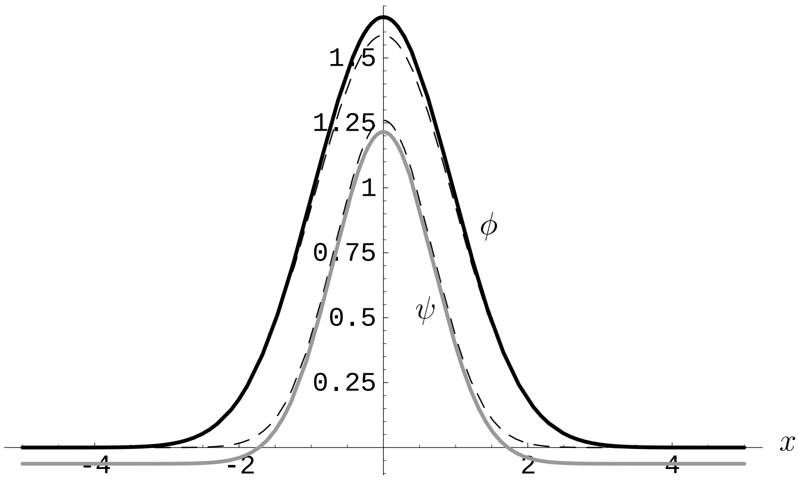

Figure 2: Numerical solution for the double lump of codimension one, with $p = 2$ and $\lambda = 1/2$. The thick black line is $\phi(x)$ and the thick gray line is $\psi(x)$. For comparison, we show, in dashed lines, the solutions for $\lambda = 0$. We note that $\psi(x)$ tends to its asymptotic value very fast, This is in contrast to the case of the single lump of ϕ, which causes oscillations of ψ (see section 4). This is related to the fact that closed string modes do not propagate outside the double lump.

All the fluctuation modes can be viewed as fields propagating on our closed tachyon lump. These lumps look in all respects as spacetimes of lower dimensions, whose dynamics is governed by some noncritical string theory. The fact that we cannot put on these closed lumps open lumps of higher dimension, and that the excitation modes of the open lumps we can put in cannot escape in the transverse direction, is only confirming our interpretation.

The statement that the fluctuations of the background are confined to the lump is tantamount to the fact that $\delta\phi(y, x)$ are normalizable in the x direction. Further support to our claim that none of the modes of the p-adic closed string propagates outside the closed tachyon lump, comes from numerical study[4] of the double lump solution at finite coupling λ. It is clear on our numerical solution in figure 2 for $p = 2$ and $\lambda = 1/2$ that both fields approach their true vacuum very fast. This is unlike the case of back-reaction of a static D-brane in the perturbative closed string vacuum $\psi = 1$, which causes spatial oscillations of the closed string tachyon far away from the lump as we will show in section 4.

4. Back-reaction of static D-branes

In this section we shall study how static open string lumps backreact on the closed string tachyonic background. Later in section 5 we will generalize these results to decaying time dependent lumps. In general finding an exact solution to the full equations of motion of a realistic open-closed string field theory is a rather hard problem. In p-adic string theory the full equations of motion (3.1) are much simpler, but still too complicated to admit a

[4] To solve (3.1) numerically, we use iterations of the convolution formulae (see [13] and [22]), starting with the solution at zero coupling (3.5) and (3.6). For lumps, however, the iteration method by itself is not convergent. But we can fix this by rescaling the solutions at each step: $\phi(x) \to \alpha\phi(x)$ and $\psi(x) \to \beta\psi(x)$, where α and β can be determined from the integral over x of the equations of motion (3.1).

simple analytic solution for finite values of λ, which is proportional to the string coupling g. We shall start by writing a perturbative expansion for the open and closed tachyon fields

$$\psi = \psi_0 + \lambda^2 \psi_1 + O(\lambda^4)\,, \tag{4.1}$$
$$\phi = \phi_0 + \lambda^2 \phi_1 + O(\lambda^4)\,. \tag{4.2}$$

As our starting point we take a solution describing a codimension-d D-brane in a flat background

$$\psi_0(x,y) = 1\,, \tag{4.3}$$
$$\phi_0(x,y) = p^{\frac{d}{2(p-1)}}\, e^{-\frac{1}{2\log p}\frac{p-1}{p}x^2} \tag{4.4}$$

and we will look for the first order correction ψ_1 describing the back-reaction. Here x is the d-dimensional transverse coordinate to the D-brane and y is the coordinate along its worldvolume. To find $\phi_0(x,y)$ we have made use of the identity (3.4). The equation for ψ_1 looks as

$$p^{-\frac{1}{4}\Box}\psi_1 = p^2\psi_1 + \frac{p-1}{2p}\left(\phi_0^{p+1} - 1\right). \tag{4.5}$$

Solutions to the homogeneous part of this equation are plane waves e^{ikx} with

$$k^2 = 8 + \frac{4}{\log p}\cdot 2\pi i n \tag{4.6}$$

parametrized by an arbitrary integer n. These values of k^2 appear of course also as poles in the Green's function, they give us the particle content of the theory. The lowest mode has mass squared $m^2 = -8$ and is the closed string tachyon. The higher modes have complex mass squared and are a bit mysterious. One can argue that in the adelic theory [6] these particles disappear and the true closed string spectrum appears. The only fact we will need, which is true for either theory is that the only particle with mass m lying on (or very close to) the imaginary axis is the tachyon.

Since the unperturbed lumps are spherically symmetric in the transverse dimension, we concentrate now only on spherically symmetric solutions of the homogeneous part of the equation. The solutions can be readily found in terms of the Bessel functions, one of them being

$$\psi_{1\,homog.} \sim r^{-\frac{d-2}{2}} J_{\frac{d-2}{2}}\left(\sqrt{8}r\right). \tag{4.7}$$

A second solution can be obtained by changing the Bessel function J_ν into Y_ν, it is singular at the origin for $d > 1$ however. To find a particular solution of (4.5) we first get rid of the constant part by writing

$$\psi_1(x) = \frac{1}{2p(p+1)} + \tilde{\psi}_1(x) \tag{4.8}$$

and pass to the Fourier transform

$$\tilde{\psi}_1(x) = \int d^dk\, e^{ikx}\tilde{\psi}_1(k)\,. \tag{4.9}$$

Now the equation (4.5) takes the form

$$p^{\frac{1}{4}k^2}\tilde{\psi}_1(k) = p^2\tilde{\psi}_1(k) + \frac{p-1}{2p}\left(\frac{p^{\frac{2p}{p-1}}\log p}{2\pi(p^2-1)}\right)^{\frac{d}{2}} e^{-\frac{p\log p}{2(p^2-1)}k^2} \tag{4.10}$$

and therefore a particular solution is given by

$$\tilde{\psi}_1(x) = \frac{p-1}{2p}\left(\frac{p^{\frac{2p}{p-1}}\log p}{2\pi(p^2-1)}\right)^{\frac{d}{2}} \int d^dk\, e^{ikx}\frac{1}{p^{\frac{1}{4}k^2}-p^2}\, p^{-\frac{p}{2(p^2-1)}k^2} . \tag{4.11}$$

While this integral is divergent on a codimension one hypersurface $k^2 = 8$, it can be assigned a principal value. Passing to the spherical coordinates, the integral can be rewritten as

$$S_{d-2}\int_0^\infty dk\, k^{d-1}\int_0^\pi d\theta\,(\sin\theta)^{d-2}\, e^{ikr\cos\theta}\frac{1}{p^{\frac{1}{4}k^2}-p^2}\, p^{-\frac{p}{2(p^2-1)}k^2} , \tag{4.12}$$

where S_{d-2} is the volume of a $d-2$ dimensional sphere. The integral over the angular variable is

$$\int_0^\pi d\theta(\sin\theta)^{d-2}\, e^{ikr\cos\theta} = \sqrt{\pi}\,\Gamma\left(\frac{d-1}{2}\right)\left(\frac{2}{kr}\right)^{\frac{d-2}{2}} J_{\frac{d-2}{2}}(kr) , \tag{4.13}$$

the expression (4.12) now becomes

$$S_{d-2}\sqrt{\pi}\,\Gamma\left(\frac{d-1}{2}\right)\left(\frac{2}{r}\right)^{\frac{d-2}{2}}\int_0^\infty dk\, k^{\frac{d}{2}} J_{\frac{d-2}{2}}(kr)\frac{1}{p^{\frac{1}{4}k^2}-p^2}\, p^{-\frac{p}{2(p^2-1)}k^2} . \tag{4.14}$$

To study the large r behavior, we use the relation between the Bessel functions $J_\nu(z)$, $Y_\nu(z)$ and the Hankel functions $H_\nu^{(1,2)}$

$$J_\nu(z) = \frac{1}{2}\left(H_\nu^{(1)} + H_\nu^{(2)}\right) , \tag{4.15}$$

$$Y_\nu(z) = \frac{1}{2i}\left(H_\nu^{(1)} - H_\nu^{(2)}\right) . \tag{4.16}$$

Splitting the integral (4.14) into two parts containing the Hankel functions $H_\nu^{(1)}$ and $H_\nu^{(2)}$, we add a small semi-circle contour to each of the principal value integrals, such that in both cases the integration will be along a single contour, bypassing the pole at $k = \sqrt{8}$ from above or below respectively. These contours can then be further deformed to straight lines going from the origin to infinity in a small finite angle to the real axis, requiring only that the contours do not cross any of the other complex poles. The integrals along these two lines are finite and regular and one can take a limit $r \to \infty$ in both of them separately. Using the known asymptotic behavior of the Hankel functions, both integrals exhibit an exponential decay for large r. Therefore the only exponentially unsuppressed contribution comes from the two small semi-circles around the pole. Evaluating these half-residua we find

$$S_{d-2}\sqrt{\pi}\,\Gamma\left(\frac{d-1}{2}\right)\left(\frac{2}{r}\right)^{\frac{d-2}{2}} Y_{\frac{d-2}{2}}(\sqrt{8}r)\,\frac{(-\pi)8^{\frac{d}{4}}}{\sqrt{2}p^2\log p}\, p^{-\frac{4p}{(p^2-1)}} . \tag{4.17}$$

Finally the most general spherically symmetric solution to (4.5) behaves for large r as

$$\psi_1(r) \approx \frac{1}{2p(p+1)} + Ar^{-\frac{d-2}{2}} Y_{\frac{d-2}{2}}(\sqrt{8}r) + Br^{-\frac{d-2}{2}} J_{\frac{d-2}{2}}(\sqrt{8}r)\,, \tag{4.18}$$

where

$$A = -\frac{\pi}{\sqrt{8}} \left(\frac{\sqrt{8}\, p^{\frac{2p}{p-1}} \log p}{(p^2-1)} \right)^{\frac{d}{2}} \frac{p-1}{p^3 \log p}\, p^{-\frac{4p}{(p^2-1)}} \tag{4.19}$$

and B is an arbitrary constant. Note that the constant part in (4.18) accounts for the fact, that in the true open string vacuum with $\phi = 0$, the value of the closed string tachyon ψ in its perturbative vacuum is shifted precisely by this constant times λ^2 plus higher order corrections.

Now one may ask whether there are some natural boundary conditions on the field $\psi(x)$, which would fix the value of the parameter B and produce thus a unique answer for the perturbed background. For large r both the particular and the homogeneous solutions exhibit the same kind of oscillations, they differ only in the relative phase and by an arbitrary constant scale factor for the amplitude. There is no way how one could fix uniquely this parameter. The physical reason is that this ambiguity reflects the genuine instability of the tachyonic background.

5. Time dependent solutions

We shall now turn our attention to the time dependent solutions of the coupled equations of motion (3.1). Physically this is equivalent to the study of closed strings emission from decaying D-branes [17, 19, 21]. We will be interested here in those solutions in which both open and closed tachyon fields were in their perturbative vacua in the infinite past. We will refer to those solutions as half-S-branes [18]. As in [13] we are led to the following ansatz:

$$\begin{aligned} \phi(t) &= 1 - \sum_{n=1}^{\infty} a_n e^{\alpha n t}\,, \\ \psi(t) &= 1 - \sum_{n=1}^{\infty} b_n e^{\beta n t}\,, \end{aligned} \tag{5.1}$$

assuming that $\alpha, \beta > 0$. The expansion in exponentials is very natural from the point of view of string field theory [10]. One might expect at first, that $\alpha = \sqrt{2}$ and $a_1 \neq 0$ describes open string tachyon condensation, whereas $\beta = \sqrt{8}$ and $b_1 \neq 0$ describes closed string tachyon condensation. Closer inspection reveals that indeed this is true to the zeroth order in the coupling λ. At higher orders one has to take $\alpha = \beta$ and further equal to one of the two possible values

$$\begin{aligned} \alpha_1 &= \sqrt{8} + \frac{p^2-1}{4\sqrt{2}\, p^3 (1+p+p^2) \log p} \lambda^2 + O(\lambda^4)\,, \\ \alpha_2 &= \sqrt{2} - \frac{(p^2-1)(1+p\sqrt{p})}{4\sqrt{2}\, p^{\frac{3}{2}} (1+p+p^2) \log p} \lambda^2 + O(\lambda^4)\,. \end{aligned} \tag{5.2}$$

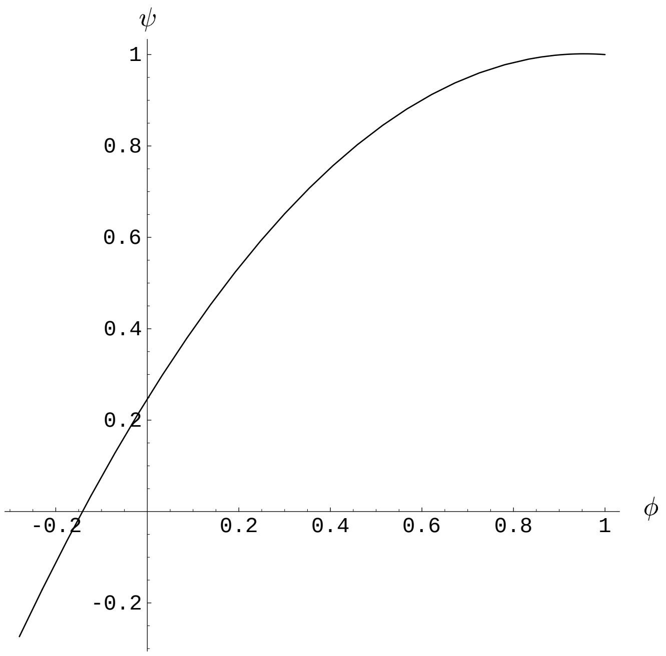

Figure 3: Time evolution of the coupled open-closed tachyon system drawn in the (ϕ, ψ) plane. The system starts its evolution in the infinite past at the point $(1, 1)$, reaching the point $\phi = 0$ around $t \sim 0$. For this picture we have chosen $p = 2$ and $\lambda = 0.001$, to indicate that even at very small coupling the exponential ansatz (5.1) leads to condensation of both tachyons.

What might be more surprising is, that a_1 and b_1 cannot be chosen arbitrarily, their ratio $x = b_1/a_1$ has to be fixed to one of the values

$$\begin{aligned} x_1 &= 2(1 + p + p^2) + \frac{p^2 - 1}{p^3 - 1}\lambda^2 + O(\lambda^4)\,, \\ x_2 &= -\frac{p^2 - 1}{2(p^3 - p^{\frac{3}{2}})}\lambda^2 + O(\lambda^4) \end{aligned} \tag{5.3}$$

respectively, depending on our choice of α. The authors of [13] found a one parameter family of solutions which corresponds to translations in time. It has one parameter less than the Sen's solutions [10], due to the fact that we require our tachyon to be in its unstable vacuum in the far past.

For the two tachyon fields $\phi(t)$ and $\psi(t)$ one would expect a two parameter family of solutions, but the solutions described by the ansatz (5.1) have only one free parameter. Although it may look like we have two discrete choices for b_1/a_1 corresponding to either open or closed tachyon condensation, numerical investigation reveals that the two processes are mixed together. Solving recursively for the coefficients a_n, b_n with the initial conditions corresponding to the rolling of the open string tachyon, we find the time evolution of both fields $\phi(t)$ and $\psi(t)$, which we plot in the ϕ, ψ plane in figure 3. Although at very early times the open string tachyon changes much faster than the closed one, by the time the open string tachyon reaches zero, the closed one is also quite close to zero, suggesting that actually both tachyons condense. Following the evolution further on, beyond what is shown in figure 3, we find the wild oscillations familiar from [13], whose significance is not yet entirely understood.

Another strategy for finding exact solutions to the coupled equations of motion would be using perturbation theory as in section 4. This will eventually lead to the correct two parameter family of solutions. Let us describe first the solution where only the open string tachyon condenses. We take $\psi_0 = 1$ and

$$\phi_0(t) = 1 - e^{\sqrt{2}t} + \frac{1}{2(p^2+p+1)} e^{2\sqrt{2}t} + \cdots . \tag{5.4}$$

being the rolling solution in open p-adic string theory studied in [13]. Solving for ψ_1 from (4.5) we find

$$\psi_1(t) = \frac{p^2-1}{2p(p^2-\sqrt{p})} e^{\sqrt{2}t} + \frac{(p^2-1)^2(p^2+1)}{4\sqrt{2}\,p^3(p^3-1)\log p} t e^{2\sqrt{2}t} + \sum_{n=3}^{\infty} c_n e^{n\sqrt{2}t} , \tag{5.5}$$

where c_n are easily calculable p-dependent constants. We can of course add to (5.5) an arbitrary multiple of $e^{2\sqrt{2}t}$. What is a bit surprising, is the explicit appearance of the factor t in the second term of (5.5). It stems from the fact that the closed string tachyon mass is exactly twice the mass of the open one. For special values of the initial conditions, all the terms at higher orders in λ with explicit polynomial factors in t combine into the corrected exponentials of (5.2). Note that those terms violate the periodicity in the imaginary time direction

$$t \to t + \sqrt{2}\pi i . \tag{5.6}$$

One might think that the wild oscillations of both open and closed tachyon fields are related to the fact that we were dealing with translationally invariant solutions, corresponding to the space-filling D-brane. It is therefore interesting to look what happens to the decay of a D0-brane, where one can imagine closed string "radiation" carrying away all the energy. One can again use the ansatz (5.1) with a_n and b_n now being functions of x. The equations for those coefficients are no longer algebraic, instead they contain factors like $p^{-\Box/2}$. Analytically we can find the large distance behavior of the first order correction in λ. This is analogous to calculating the emission of gravitational waves from a non static source in general relativity.

Practically we are going to solve the equation (4.5) with $\phi_0(x,t)$ being now a product of the spatial lump $\phi_0^{lump}(x)$ and the rolling solution $\phi_0^{roll}(t)$ of [13]. Expanding $\psi_1(x,t)$ into powers of $e^{\sqrt{2}\,t}$ and solving for the leading large distance behavior of the individual coefficients as in section 4 we find

$$\psi_1(r,t) \approx \frac{1}{2p(p+1)} + \sum_{n=0}^{\infty} A_n e^{\sqrt{2}\,nt} r^{-\frac{d-2}{2}} Y_{\frac{d-2}{2}}(r\sqrt{8-2n^2}) + B_n e^{\sqrt{2}\,nt} r^{-\frac{d-2}{2}} J_{\frac{d-2}{2}}(r\sqrt{8-2n^2}) , \tag{5.7}$$

where A_n are finite calculable constants dependent on p and d, and B_n are again free parameters. We are getting thus true spatial oscillations (up to a power in r) for $n = 0, 1$. Terms with $n > 2$ are superluminal radial waves with speeds ranging between $3/\sqrt{5} \approx 1.34$ and 1 the speed of light. This should not be too surprising since we are dealing with closed

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string tachyon waves. One might hope, that resumming the series (5.7) one would get rid of the wild oscillations in time, but that is unfortunately beyond our capabilities to prove or disprove.

It would be interesting to have a well defined splitting of the energy between the open and closed string sector. Whereas we can confirm that the total energy is conserved for the solution shown on figure 3, we are unable to split it in the two sectors in any meaningful way. This problem is certainly familiar from general relativity, where one can achieve this goal under very special circumstances. In that case one can argue, that the difference between constant ADM energy and time dependent Bondi energy measures the energy emitted in the gravitational waves.[5] Both ADM and Bondi energy are defined by integrals over the metric at spatial infinity or $\mathcal{I}^+$ respectively. Since in p-adic string theory the closed string tachyon has different properties than the metric in general relativity, it seems hard to find any useful analogue of the Bondi energy in the p-adic string theory.

Some other solutions. In purely open p-adic string theory, it seems very unlikely that there exist lump solutions in the time direction. In fact, it has been shown in [13] that, when p is even, there can be no monotonic lump (a tachyon rolling monotonically from a value $\phi = b$ towards the true vacuum until it reaches some value $a < b$, then rolling back to $\phi = b$ monotonically).

Here, we want to show that in open-closed p-adic string theory, open tachyon lumps in time do exist as long as we allow the closed tachyon field to diverge in the far past and far future. Unfortunately we are not able to solve the equations of motion with a nonzero coupling,[6] we will thus only consider $\lambda = 0$.

The equations of motion (3.1) admit an interesting solution obtained by Wick-rotating the spatial lump (3.5)

$$\psi(t) = p^{\frac{1}{p^2-1}} \exp\left(\frac{p^2-1}{p^2 \log p}\, t^2\right). \tag{5.8}$$

Plugging this into (3.1) and trying the ansatz $\phi(t) = Ae^{-b^2t^2}$, we find

$$\phi(t) = p^{-\frac{2p+1}{2(p^2-1)}} \exp\left(-\frac{p-1}{2\log p}\, t^2\right). \tag{5.9}$$

For this solution, the open tachyon is in its true vacuum in the far past and in the far future, and in the vicinity of its perturbative vacuum only for a short time around $t = 0$. One might be tempted to interpret this solution as an S-brane [9]. We should be very cautious though, because the solution requires rather special "cosmological" circumstances and moreover lacks the periodicity in imaginary time.

[5] For recent application of these ideas to the study of closed string tachyon condensation, see [23].

[6] We can solve the equations of motion numerically when the fields at infinity converge fast enough to constant values (like in the case of the double lump). But here we will see that $\psi(t)$ diverges at infinity, and we are not able to construct a numerical solution.

6. Discussions and conclusions

Although the relation of p-adic string theory to more conventional string theories remains rather unclear, the theory has proven to be a good toy model for studies of tachyon condensation. One of the surprising features of the closed p-adic strings is that they look very similar to the open p-adic strings. In fact, both theories contain only a tachyon with a peculiar nonlocal interaction. In the case of open strings there are known lump solutions, whose tensions and excitation spectra give them the interpretation of D-branes. We found analogous solutions in the closed string sector as well. Based on their lump-like nature and excitation spectra, we have argued that they correspond to spacetimes of lower dimensions. Outside the core of the lump, the tachyon field is in its true vacuum and one can easily check that there are indeed no propagating perturbative degrees of freedom. Further support to our claims comes from the study of possible open string lumps, i.e. D-branes, which can be put in this closed string background. Far from the core of the closed string lump, the open string tachyon has to be in its true vacuum. Therefore the D-brane dimension has to be always smaller or equal to that of the spacetime.

It is very tempting to speculate that similar closed string tachyon lumps exist in realistic nonsupersymmetric string theories. All physics including gravity would be localized on these lumps, and they would thus form a viable alternative to compactification or large extra dimensions scenarios.

The second main problem we dealt with is the dynamics of the open string tachyon condensation. The p-adic model is simple enough to allow us to solve for the backreaction of open tachyon configurations on the closed string background. In particular, we have given the analytic form of the fluctuations of the closed tachyon far away from a static or decaying D-brane. Although we hoped that a p-adic string theory including the closed string sector would have nicer time dependent solutions than pure open p-adic string theory, that doesn't seem to be the case. Our rolling solutions, calculated in a series expansion scheme, do in fact oscillate with diverging amplitudes. One could still hope, however, that including higher orders in the coupling constant might tame these oscillations. Further investigation is needed on this issue. A slightly disappointing fact which should have been expected, is that we were not able to separate the expression for the total energy into separate open and closed sectors; such a separation would be useful to see the dissipation of the rolling tachyon's energy into the closed modes. This is certainly reminiscent of the situation in general relativity.

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